Write your name here

Surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number | Candidate Number

Further Pure Mathematics FP1

Advanced/Advanced Subsidiary

Wednesday 29 January 2014 – Morning
Time: 1 hour 30 minutes

Paper Reference
6667A/01

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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1. $f(x) = 2x - 5\cos x$, where x is in radians.

(a) Show that the equation $f(x) = 0$ has a root α in the interval $[1, 1.4]$.

(2)

(b) Starting with the interval $[1, 1.4]$, use interval bisection twice to find an interval of width 0.1 which contains α.

(3)

Leave blank

Question 1 continued

Q1

(Total 5 marks)

P 4 3 0 1 9 A 0 3 3 2

Turn over

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2.

(i) $$\mathbf{A} = \begin{pmatrix} -4 & 10 \\ -3 & k \end{pmatrix}, \quad \text{where } k \text{ is a constant.}$$

The triangle T is transformed to the triangle T' by the transformation represented by **A**.

Given that the area of triangle T' is twice the area of triangle T,
find the possible values of k.

(4)

(ii) Given that

$$\mathbf{B} = \begin{pmatrix} 1 & -2 & 3 \\ -2 & 5 & 1 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 2 & 8 \\ 0 & 2 \\ 1 & -2 \end{pmatrix}$$

find **BC**.

(3)

Leave blank

Question 2 continued

(Total 7 marks)

Q2

Leave blank

3. A rectangular hyperbola has parametric equations

$$x = 2t, \quad y = \frac{2}{t}, \qquad t \neq 0$$

Points P and Q on this hyperbola have parameters $t = \frac{1}{2}$ and $t = 4$ respectively.

The line L, which passes through the origin O, is perpendicular to the chord PQ.

Find an equation for L.

(4)

P 4 3 0 1 9 A 0 6 3 2

Leave blank

Question 3 continued

Q3

(Total 4 marks)

P 4 3 0 1 9 A 0 7 3 2

Turn over

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4. $$\mathrm{f}(x) = 2x^{\frac{1}{2}} - \frac{6}{x^2} - 3, \quad x > 0$$

A root β of the equation $\mathrm{f}(x) = 0$ lies in the interval [3, 4].

Taking 3.5 as a first approximation to β, apply the Newton-Raphson process once to $\mathrm{f}(x)$ to obtain a second approximation to β. Give your answer to 3 decimal places.

(5)

P 4 3 0 1 9 A 0 8 3 2

Leave blank

Question 4 continued

Q4

(Total 5 marks)

Turn over

P 4 3 0 1 9 A 0 9 3 2

Leave blank

5. $$z = 5 + \mathrm{i}\sqrt{3}, \qquad w = \sqrt{3} - \mathrm{i}$$

(a) Find the value of $|w|$. **(1)**

Find in the form $a + \mathrm{i}b$, where a and b are real constants,

(b) zw, showing clearly how you obtained your answer, **(2)**

(c) $\frac{z}{w}$, showing clearly how you obtained your answer. **(3)**

Given that

$$\arg(z + \lambda) = \frac{\pi}{3}, \qquad \text{where } \lambda \text{ is a real constant,}$$

(d) find the value of λ. **(2)**

Leave blank

Question 5 continued

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Question 5 continued

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Question 5 continued

Q5

(Total 8 marks)

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6. (a) Use the standard results for $\sum_{r=1}^{n} r^3$ and $\sum_{r=1}^{n} r$ to show that for all positive integers n,

$$\sum_{r=1}^{n} r(r+1)(r-1) = \frac{1}{4}n(n+1)(n-1)(n+a)$$

where a is an integer to be determined.

(4)

(b) Hence find the value of n, where $n > 1$, that satisfies

$$\sum_{r=1}^{n} r(r+1)(r-1) = 10\sum_{r=1}^{n} r^2$$

(5)

Leave blank

Question 6 continued

P 4 3 0 1 9 A 0 1 5 3 2

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Question 6 continued

Leave blank

Question 6 continued

Q6

(Total 9 marks)

P 4 3 0 1 9 A 0 1 7 3 2

Leave blank

7. $$\mathbf{P} = \begin{pmatrix} 3a & -2a \\ -b & 2b \end{pmatrix}, \quad \mathbf{M} = \begin{pmatrix} -6a & 7a \\ 2b & -b \end{pmatrix}$$

where a and b are non-zero constants.

(a) Find $\mathbf{P}^{-1}$, leaving your answer in terms of a and b.

(3)

Given that

$$\mathbf{M} = \mathbf{PQ}$$

(b) find the matrix **Q**, giving your answer in its simplest form.

(3)

Leave blank

Question 7 continued

Q7

(Total 6 marks)

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8. The parabola C has equation $y^2 = 4ax$, where a is a positive constant.

The point $P(ap^2, 2ap)$ lies on the parabola C.

(a) Show that an equation of the normal to C at P is

$$y + px = ap^3 + 2ap$$

(5)

The normal to C at the point P meets the x-axis at the point $(6a, 0)$ and meets the directrix of C at the point D. Given that $p > 0$,

(b) find, in terms of a, the coordinates of D.

(4)

Given also that the directrix of C cuts the x-axis at the point X,

(c) find, in terms of a, the area of the triangle XPD, giving your answer in its simplest form.

(3)

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Question 8 continued

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Question 8 continued

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Question 8 continued

Q8

(Total 12 marks)

P 4 3 0 1 9 A 0 2 3 3 2

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9. Given that $z = x + iy$, where $x \in \mathbb{R}$, $y \in \mathbb{R}$, find the value of x and the value of y such that

$$(3 - i)z^* + 2iz = 9 - i$$

where z^* is the complex conjugate of z.

(8)

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Question 9 continued

P 4 3 0 1 9 A 0 2 5 3 2

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Question 9 continued

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Question 9 continued

Q9

(Total 8 marks)

P 4 3 0 1 9 A 0 2 7 3 2

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10. (i) A sequence of numbers $u_1, u_2, u_3, \ldots$, is defined by

$$u_{n+1} = 5u_n + 3, \quad u_1 = 3$$

Prove by induction that, for $n \in \mathbb{Z}^+$,

$$u_n = \frac{3}{4}(5^n - 1)$$

(5)

(ii) Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\text{f}(n) = 5(5^n) - 4n - 5 \text{ is divisible by 16.}$$

(6)

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Question 10 continued

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Question 10 continued

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Question 10 continued

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Question 10 continued

Q10

(Total 11 marks)

TOTAL FOR PAPER: 75 MARKS

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